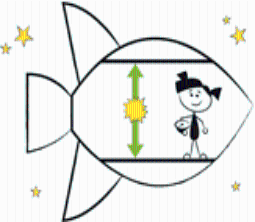
**The Twin Paradox**

The twin paradox is a famous thought experiment in special relativity. It turns out, like many so-called paradoxes, it is not a paradox at all.

In this worksheet, we will examine the twin paradox and its resolution. To set the stage, we will consider two fraternal twins, Alice and Bob. Bob is staying on planet Earth and Alice is travelling to a distant exoplanet 5.2 light years away. Alice travels at a speed of ****c. As soon as she gets to the planet, she returns home. We will also imagine that Alice can get to the speed of ****c instantaneously. To make things simple, we will ignore the fact that she would need to accelerate and decelerate.

Since Alice is moving and moving clocks run slow she should age less than Bob (by how much?). On the other hand if we consider Alice's rest frame, she will see Bob moving away from her, so shouldn't he age less? This is the idea behind the paradox. How can both twins say the other has aged less than themselves. Let's explore this paradox in greater detail.



**Part A. Moving Clocks Run Slow**

1. According to Bob how long does it take Alice to go out to the planet and come back?
2. According to Bob what time interval will Alice measure for the same journey?
3. Bob says the planet is 5.2 ly away. What is the distance to the planet measured by Alice?
4. Draw the spacetime diagram in Bob’s reference frame including the worldlines of Bob, the planet and Alice. Include the clock tick marks of Bob’s clock and Alice’s clock.
5. On the same spacetime diagram draw lines of simultaneity for Bob. When Bob’s clock reads 4 years, what time does he say that Alice’s clock reads?
6. Now draw the lines of simultaneity for Alice. When Alice’s clock reads 2 years, what time does she say Bob’s clock reads? When Alice’s clock reads 4 years, what times does she say Bob’s clock reads?
7. Both Alice and Bob say that the other’s clock is running slow. Why?
8. Upon her return Alice is not surprised at the time difference between her clock and Bob’s. Why?

**Part B. Perspective of Travelling Twin**

1. Draw the spacetime diagram in the reference frame of Alice on her outward journey. Include Bob and the planet.
2. On the same diagram draw the worldline for Alice’s return trip.
3. Describe what the travelling twin “sees”.

**Part C. Birthday Messages**

1. Each twin decides to send a radio message on their birthday to the other. How fast will the radio message travel?
2. Draw the messages sent by Bob on the spacetime diagram drawn in Part A
3. How many messages does Alice receive on the outward journey? How many on the inward journey?
4. On the same diagram draw the messages sent by Alice.
5. How many messages does Bob receive during Alice’s outward journey? How many on the inward journey?
6. What is the ratio of signals sent and received on Alice’s inward journey according to Bob?
7. Calculate the ratio of sent and received messages, this time using the relativistic Doppler shift for message frequencies:
8. Your answers in 6 and 7 should be approximately equivalent. Why?

**Putting it All Together**

In your notebook answer the following questions:

* Explain the apparent “paradox” of the travelling and stay-at-home twin
* Is there a perfect symmetry between the two twins? Why?
* Explain why both twins see the other’s clock run slow
* Why is the twin paradox not a paradox at all?

**\*\* Extra Challenge: Complete these questions and graphs using Maple. On the LE, I will post a sample worksheet with solutions.**